# The Risks of Imperfect Knowledge: Reliability Trade-Offs in Physics-Aware Neural Networks for Climate Projections

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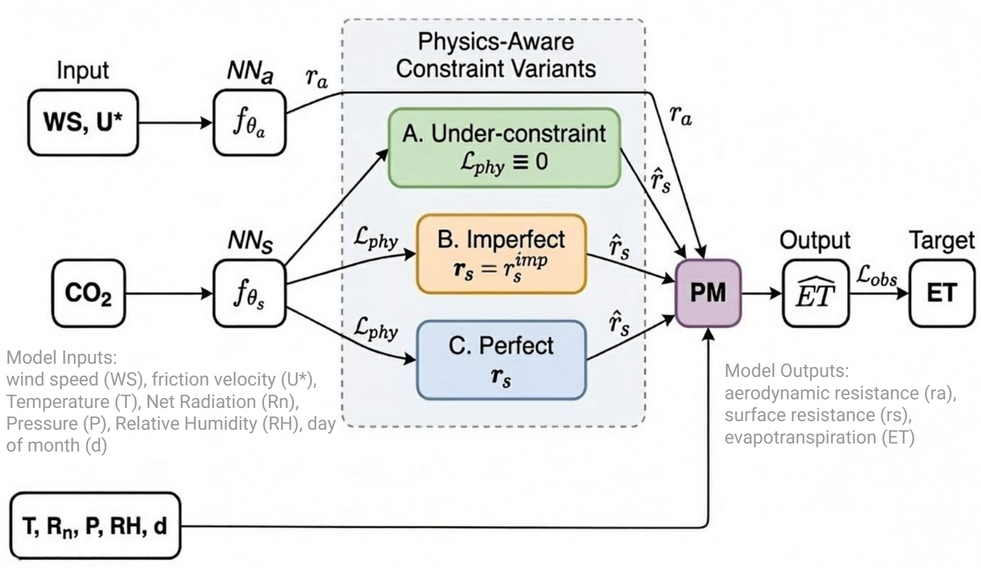
## 1. Motivation

- Hybrid models (physics + NN) promise better extrapolation, but suffer from equifinality.
- In climate projections, most equations are approximated, and uncertainties are inherent.
- A critical question: Is imperfect physical knowledge still useful in unseen regimes?

## 2. Physical Constraints & ET Projections

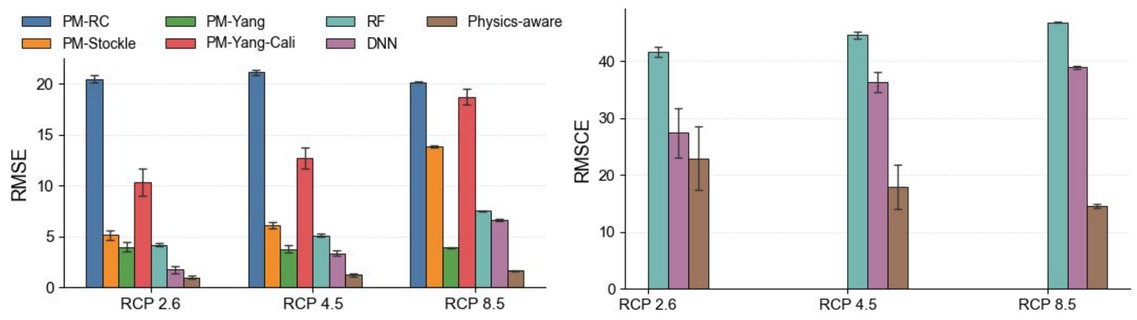



- The overall loss function of hybrid models is formulated as:

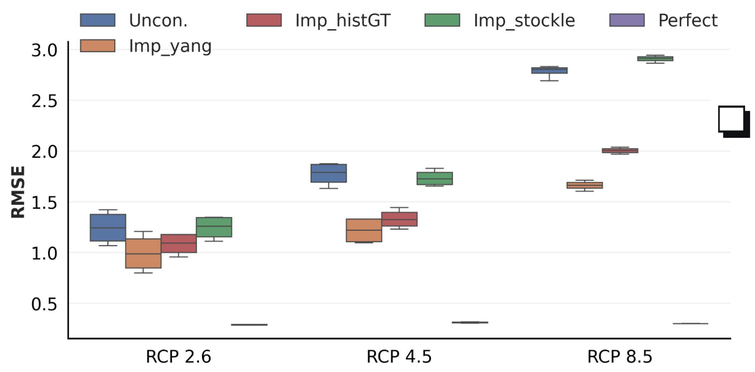
$$\mathcal{L}(\theta) = \mathcal{L}_{\text{obs}}(y, \widehat{y}) + \lambda_{\text{phy}} \mathcal{L}_{\text{phy}}(r_s, \widehat{r_s})$$

- We design three subtypes for evapotranspiration (ET) estimation: A. Under-constrained B. Imperfect C. Perfect.
- Models are trained on IPSL-CM5A-LR historical data (1966–1995) and tested under future scenarios (2006–2100) with four realisations.

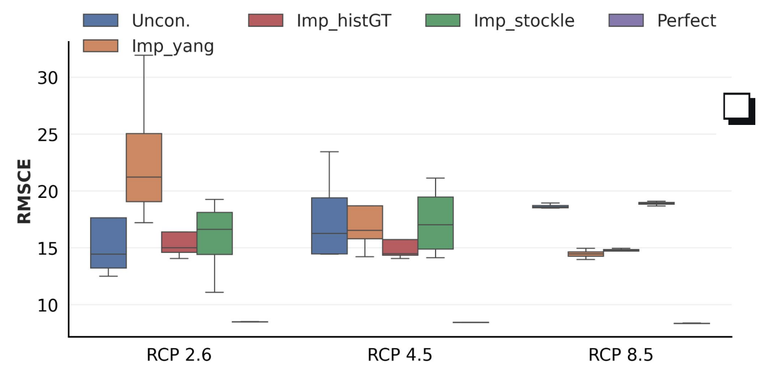
## 3. Results and Discussion



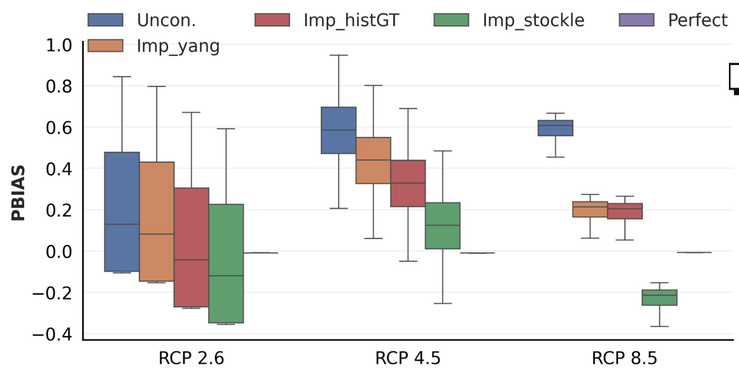

- Physics-aware NN outperforms others – but RMSE increases under RCP 8.5, and RMSCE variances rise under RCP 2.6.




- Perfect con. reduce equifinality, achieving the best.




- Imperfect con. lose reg. effect under RCP 8.5, even worse than uncon. models.




- Imperfect con. exhibit larger variance under RCP 2.6, sensitive to stochastic noises..

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## 4. Conclusion

- Imperfect con. may introduce bias that can underperform uncon. models.
- A conditional regularizer, not a reliability guarantee.
- For climate projections, extrapolation skill is regime-dependent, not structurally guaranteed.

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